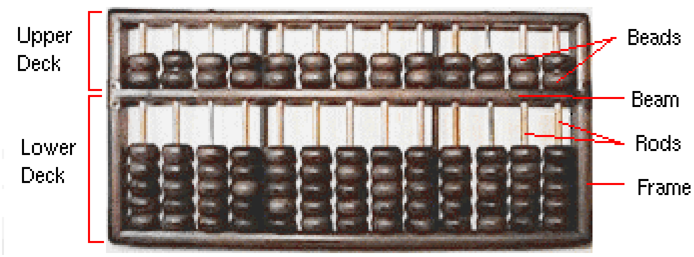
Abacus and Roman Numerals – Lab #3

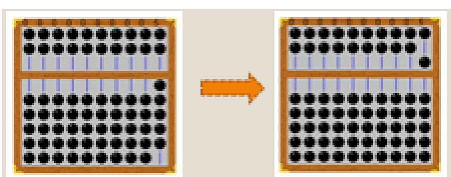
Introduction:

The abacus is a calculating tool that was used to perform arithmetic long before the written numeral system was established, and is still in use today. The labeled abacus below shows the different components of the tool:

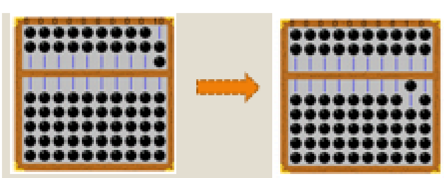


Beads in the upper deck has a value of 5, whereas beads in the lower deck have a value of 1. To count a bead, move it toward the beam that separates the two decks.

After you count 5 beads in the lower deck, you carry the result to the upper deck. Thus, both of the below representations equal the number 5:

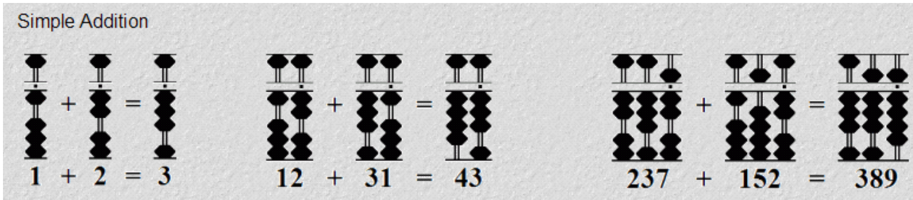


After you count both beads in the upper deck, you carry the result (10) to the left-most adjacent column. Both of the below representations equal the number 10. The left representation shows the sum of 5 "one" units + 5 "one" units, whereas the right representation shows the total of 1 "ten" unit:

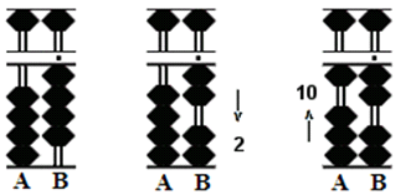


You can see in these examples that the right-most column is the ones column; the next column to the left is the tens column; the next is the hundreds column, and so on.

To see how to perform addition on an abacus, first consider the following simple examples:

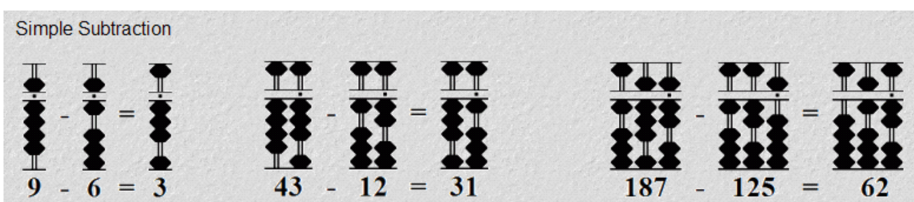


Now consider a more complex example, 4 + 8. Go through the following steps, as illustrated in the picture below:

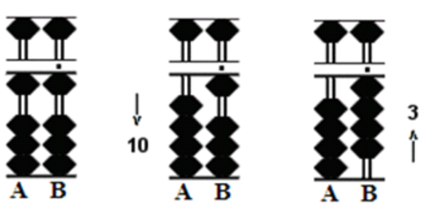


1. Set 4 on rod B
2. Add 8
    a. Rod B doesn't have 8 available, so use complementary number; the complementary number for 8 with respect to 10 is 2
    b. Subtract 2 from 4 on rod
    c. Carry 1 to tens rod A.
3. Thus, 4 + 8 = 12 becomes 4 - 2 + 10 = 12

To see how to perform subtraction on an abacus, first consider the following simple examples:



Now consider a more complex example, 11 - 7. Go through the following steps, as illustrated in the picture below:



1. Set 11 on rods AB
2. Subtract 7
    a. Since rod B only carries a value of 1, use the complement; the complementary number for 7 with respect to 10 is 3
    b. Subtract 1 from the tens rod on A
    c. Add the complementary 3 to rod B to equal 4

3. 11 - 7 = 4 becomes 11 - 10 + 3 = 4

In this lab you will also work with Roman Numerals. The values of the seven Roman Numeral symbols are presented in the table below:

Roman Numeral	Arabic Numeral
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

Instead of writing IIII for 4, the Roman Numeral system uses subtractive prefixes, so that IV is the preferred method for representing 4.

A few more complicated examples:

- XCIX = 99
  C = 100, but the preceding X is subtracted, so XC = 90
  The following IX, which = 9, is then added

- MCMXCIX = 1999
  M = 1000 and CM = 1000 - 100 = 900, so MCM = 1900
  That leaves us with XCIX: XC = 90, and IX = 9

We can perform addition using Roman Numerals using the following method:

- Uncompact subtractives

5. Concatenate symbols
6. Sort symbols from high to low
7. Group lower value symbols into higher value symbols
8. Compact subtractives

For instance, to add XIX (19) and XLV (45):

1. Uncompact subtactives: XIX becomes XVIIII and XLV becomes XXXXV
2. Concantenate: XVIIII + XXXXV becomes XVIIIIXXXXV
3. Sort: XIIIIXXXXII becomes XXXXXVVIIII
4. Group: XXXXXVVIIII becomes LXIIII
5. Compact subtractives: LXIIII becomes LXIV, the final answer of 64

We can perform subtraction using Roman Numerals using the following method:

1. Uncompact subtractives
2. Eliminate common symbols
3. Ungroup symbols

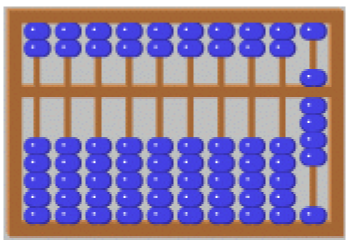
4. Eliminate common symbols
5. Repeat 3-4 until the number to be subtracted is eliminated
6. Compact subtractives

For instance, to subtract XXXVI (36) from XLIV (44):

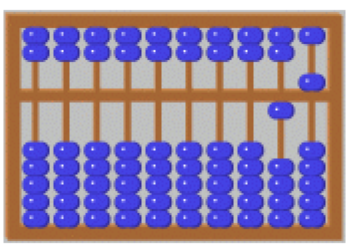
1. Uncompact subtractives: XXXVI doesn't change, but XLIV becomes XXXXIIII
2. Eliminate common symbols: XXXXIIII - XXXVI = XIII - V (the red symbols can be eliminated because they appear in both numbers)
3. Ungroup symbols: To create more common symbols, ungroup the X into VV: XIII - V = VVIII - V
4. Eliminate common symbols: VVIII - V = VIII
5. Compact subtractives: VIII doesn't change; we have a final answer of 8

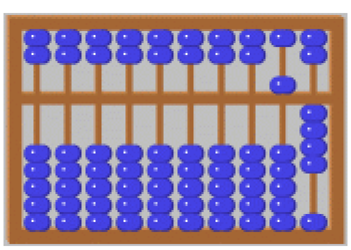
Problem Set:

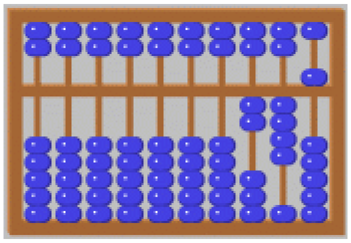
1. Convert the following abacus number representations into Roman Numerals. Do not convert to Arabic; first write the expanded Roman Numeral tally, and then compact using subtractives. Example:

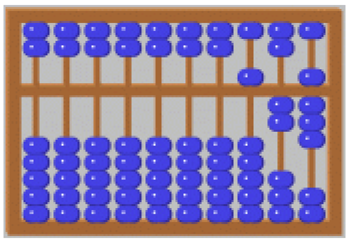


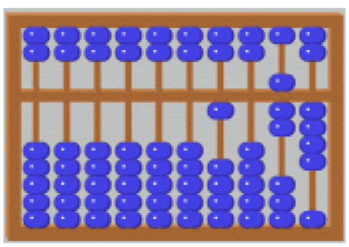
V + I + I + I + I = VIIII = IX

1. 

2. 

c. 

d. 

e. 

2. Draw an abacus representation of the following Roman Numerals. Do not convert to Arabic; first write the expanded Roman Numeral tally, then transfer to the abacus.

    a. XXIV

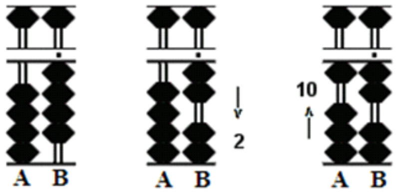
    b. CDLV

c. CMLXXXVII

d. MCDL

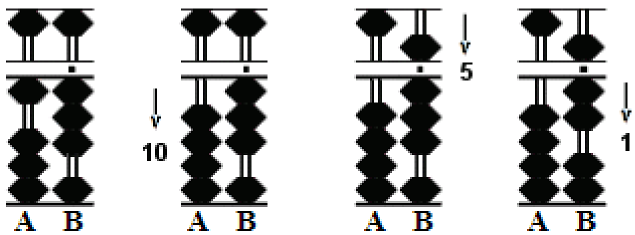
e. MMMDCIV

3. In the diagram below, determine a) what numbers are being added or subtracted, and b) how this was done using complement arithmetic. Example:



Here, 4 and 8 are being added, and this is done by subtracting 2 from 4 and then adding 10, i.e. 4 + 8 = 4 - 2 + 10

Now do the same for this diagram:



4. Explain in words, or by drawing pictures like those shown in 3, how to do the following calculations on the abacus. Include the equivalent arithmetic expression that represents the actual calculation taking place.

   Example: 9 + 7
   Put 9 in the ones column, and then add 7 by first subtracting 7's complement, 3, from the 9 in the ones column, and then adding a 10 in the tens column. So 9 + 7 = 9 - 3 + 10

   a. 6 + 6

   b. 15 + 9

   c. 36 + 75

d. 10 - 3

e. 12 - 6

f. 100 - 58

5. Do the following Roman Numeral addition problems using the Roman Numeral algorithm of uncompacting subtractives, grouping (concatenating, sorting, and combining), and compacting with subtractives.

   a. IX + VIII

   b. LII + CCXL

   c. CCCXXIII + XXXV

   d. CXCV + XXI + LXXXVIII

   e. MCMXVI + MMCCCLXXXII

6. Do the following Roman Numeral subtraction problems using the Roman Numeral algorithm of repeatedly uncompacting subtractives and eliminating common symbols, and finally compacting with subtractives.

   a. LXVIII - XII

   b. CXCII - LXIX

   c. XLII - XXXVIII

   d. CCCXCV - CXV - LXX

   e. MMDXLVII - CMLVII